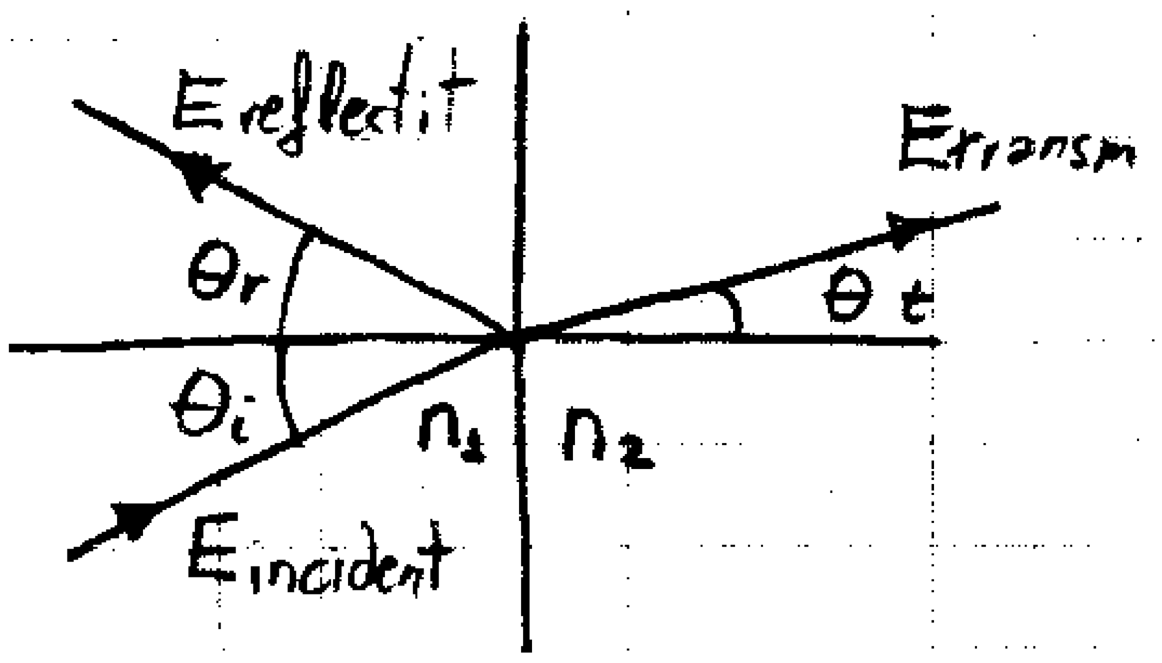
COMUNICACIONES ÓPTICAS

1-1 INTRODUCCIÓ

$$\boxed{V_{prop} = c/n}$$

Llei de reflexió $\boxed{\theta_i = \theta_r}$



Llei de Snell

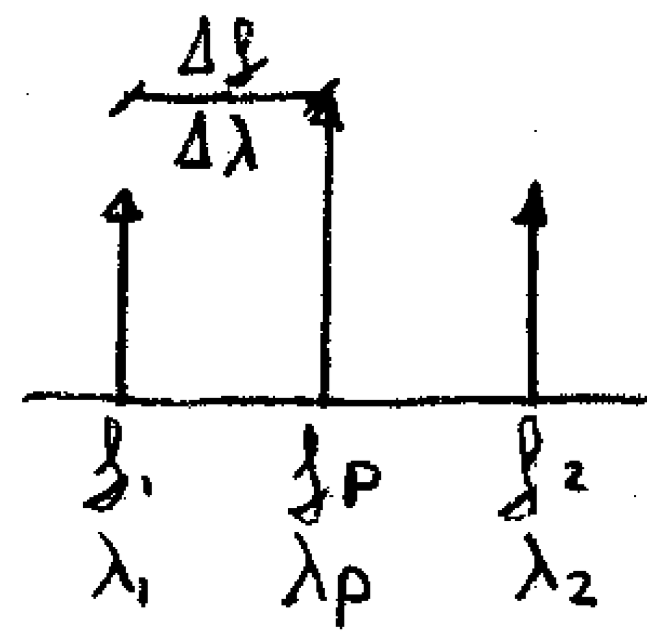
$$\boxed{n_1 \sin\theta_i = n_2 \sin\theta_t}$$

$r = \frac{E_{reflex}}{E_{incid}}$ $\boxed{R = |r|^2}$ Reflectivitat

- Reflexió total $\theta_i \geq \theta_c$ $\boxed{\theta_c = \arcsin\frac{n_2}{n_1}}$ $(n_2 < n_1)$

- Índex de refracció relatiu $\boxed{\Delta \triangleq \frac{n_1 - n_2}{n_1}}$

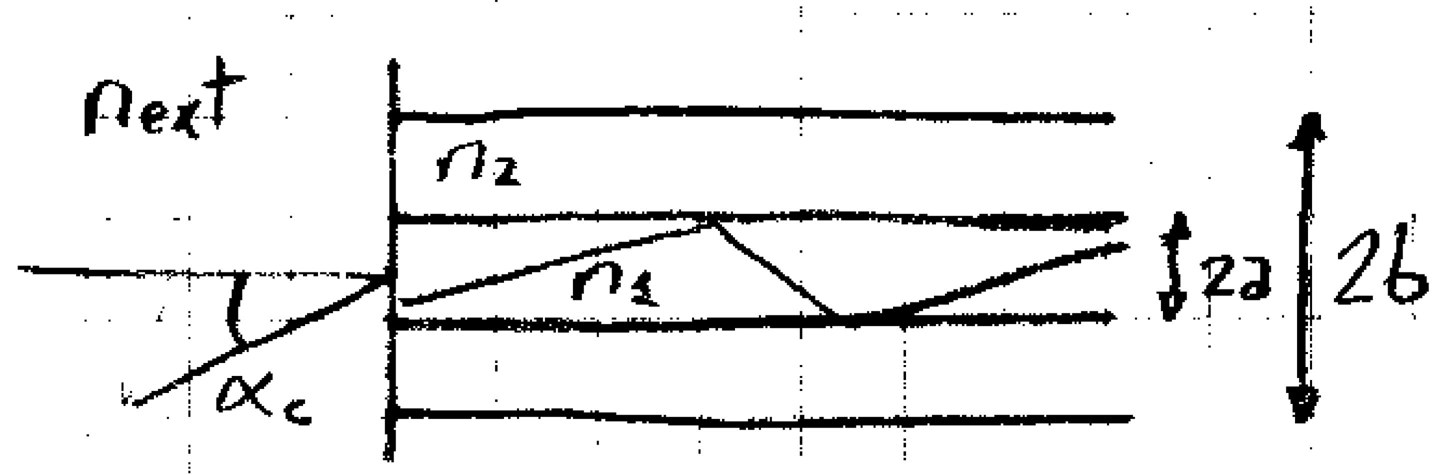
- Relació $\Delta\lambda - \Delta f$



$$\boxed{\frac{\Delta\lambda}{\lambda_p} \cong \frac{\Delta f}{f_p}}$$ $c = \lambda_p f_p$

- Atenuació $\boxed{1\,dB/m \equiv 8'686\,Nep/m}$

1-2 FIBRES ÒPTIQUES



Obertura numèrica

$$\boxed{NA \triangleq \sin\alpha_c = \frac{\sqrt{n_1^2 - n_2^2}}{n_{ext}}}$$

α_c: angle d'acceptació

- Finestres òptiques

1a	$\lambda = 850\,nm$	$\rightarrow \alpha = 2'5\,dB/Km$	Molt ample
2a	$\lambda = 1300\,nm$	$\rightarrow \alpha = 0'5\,dB/Km$	Molt estreta
3a	$\lambda = 1550\,nm$	$\rightarrow \alpha = 0'22\,dB/Km$	1550-1560 nm

- Freqüència normaltitzada

$$\boxed{V = \frac{2\pi}{\lambda} a(NA)}$$

$V > 2'405$ F.O. Multimodo

$V < 2'405$ F.O. Monomodo

- Fibres multimode de salt d'índex

\# modes $\boxed{N_t^{SI} = V^2/2}$

Dispersió intermodal $\boxed{\tau_{INTER}^{SI} \cong \frac{n_1}{c}\Delta}$ [nseg/Km]

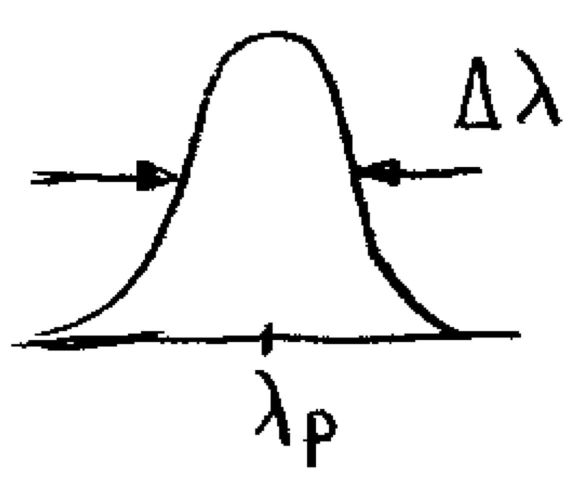
- Fibres multimode de gradient d'índex

\# modes $\boxed{N_t^{GRIN} = V^2/4}$

Dispersió intermodal $\boxed{\tau_{INTER}^{GRIN} \cong \frac{n_1}{c}\frac{\Delta^2}{8} \simeq \frac{\Delta}{8}\tau_{INTER}^{SI}}$

$\boxed{B \propto 1/\tau_{INTER}}$ [GHz·Km] $B^{GRAIN} = \frac{8}{\Delta} B^{SI}$

- Fibres monomode



velocitat de grup $\boxed{V_g = \frac{c}{n_g}}$ $\boxed{n_g \triangleq n - \lambda \frac{\partial n}{\partial \lambda}}$

Dispersió del material $\boxed{D_M \triangleq \frac{\partial}{\partial \lambda}\left(\frac{1}{V_g}\right)}$

Dispersió guia-ona $\boxed{D_w \simeq -\frac{n_{g1} - n_{g2}}{c\lambda}\frac{1'984}{V^2}}$

$\boxed{\tau_{INTRA} = \Delta\lambda |D_M + D_w|}$

- Ample de Bda $\boxed{\sigma \cdot f_0 \cong 0'1874}$

σ : Desviació tipus
f_0 : Ample de Bda per m $\boxed{\tau = 2\sigma}$

- Ample de Bda Elèctric vs Òptic

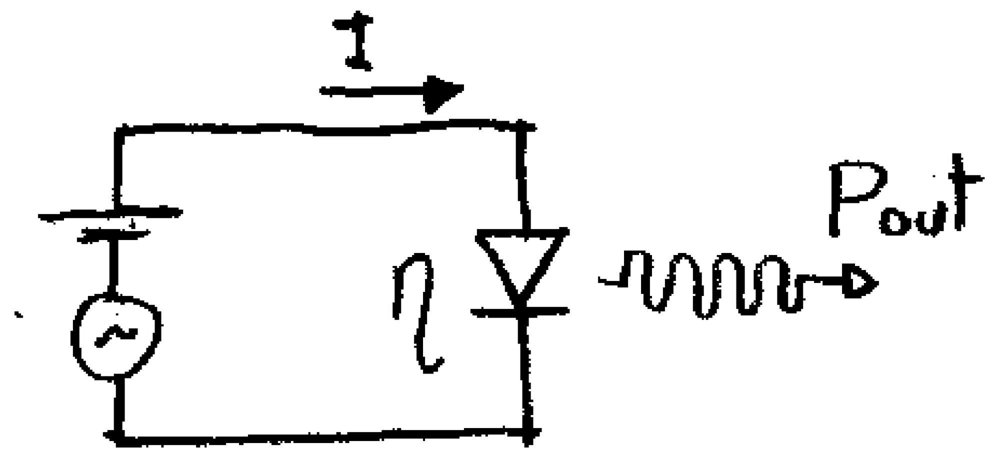
$\boxed{B_{òpt} = \sqrt{2}\, B_{elec}}$

Màxima vel. transmisió $\boxed{R_B \leq 2 B_{elec}}$

- Ample de Bda pràctic

$$B_{opt}(L) = \frac{B_{opt}(1\,Km)}{L^{\beta}}$$ (multimode) $0'5 \leq \beta \leq 1$ (monomode)

1-3 FONTS ÒPTIQUES



$$P_{out} = \eta \cdot h\nu \frac{I}{q_e}$$

η : eficiència conversió elettrò-fotò

$$<n> = \frac{P_{out}}{h\nu}$$

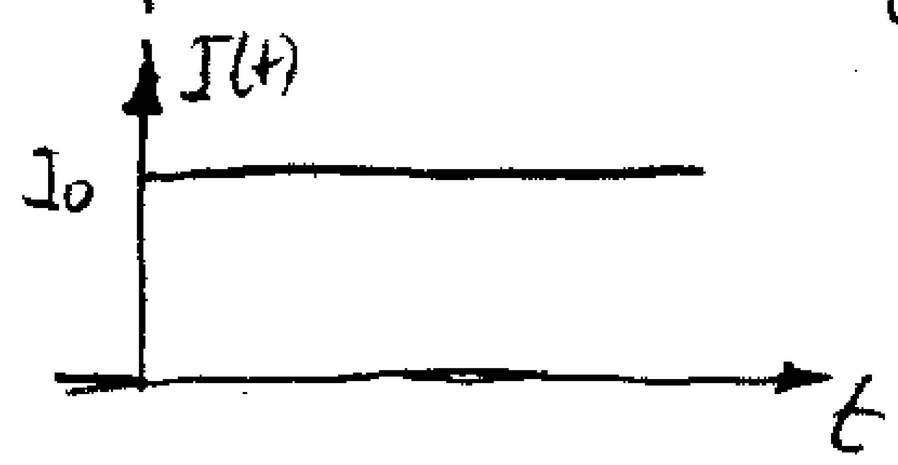
- DIODE LED

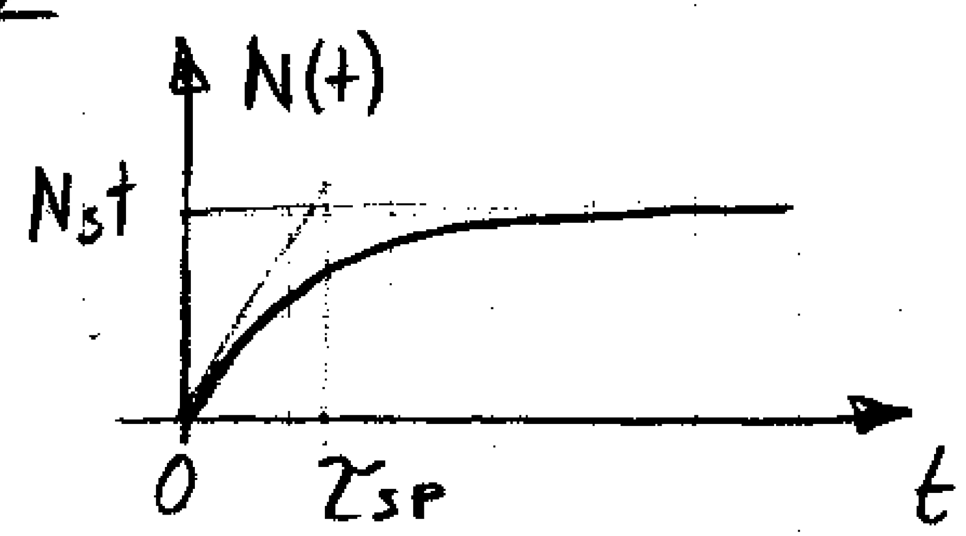
Ecuació de ritme

$$\frac{\partial N}{\partial t} = \frac{I}{qV} - \frac{N}{\tau_{sp}}$$

N : Densitat de portadors en Z.A.
τ_{sp} : Temps d'emissió espontànea

Resposta a l'esglaó





$$P_{out}(t) = \eta \frac{h\nu}{q} I_o \left(1 - e^{-t/\tau_{sp}}\right)$$

$$t_r = 2'197\,\tau_{sp}$$

$T_B \geq 3\tau_{sp}$ (conveni) $\Rightarrow$ $R_B \leq 1/3\tau_{sp}$

Modulació sinusoidal

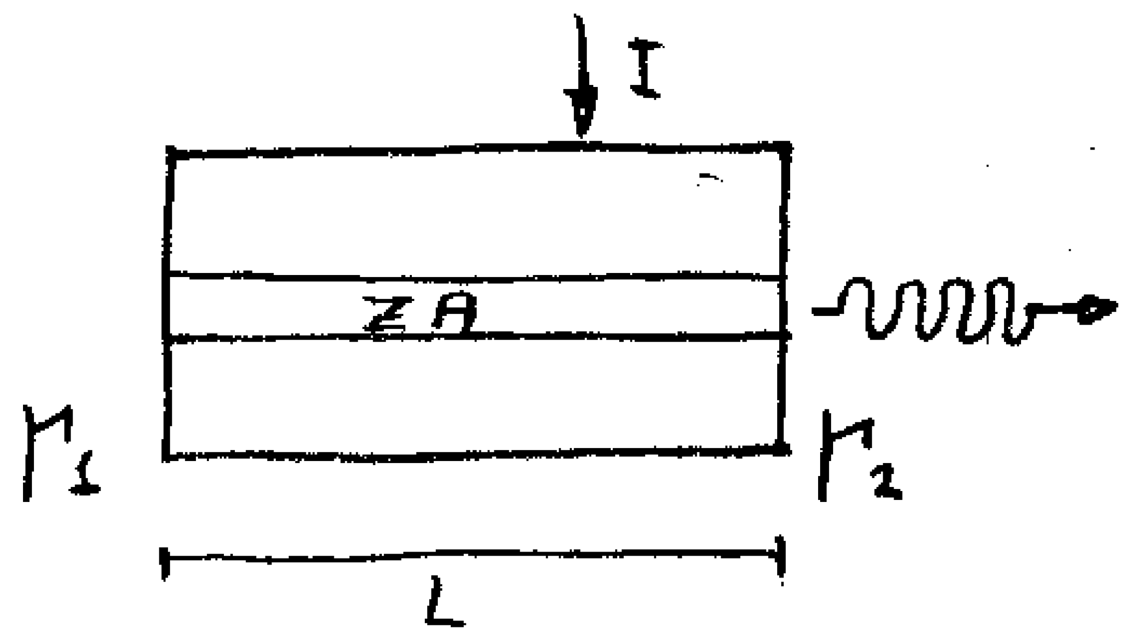
$$H(\omega) = \frac{R}{1 + j\omega\tau_{sp}} \qquad R \triangleq \eta \frac{h\nu}{q}$$

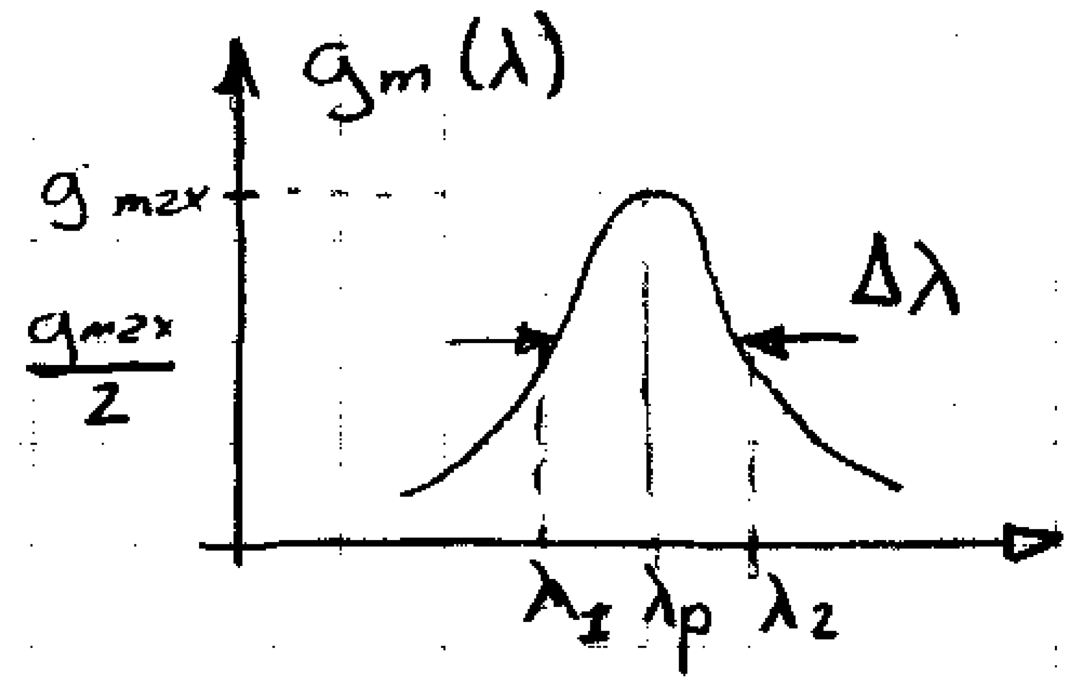
$$B_{3dB} = \frac{1}{2\pi\tau_{sp}}$$

Perdues per acoblament LED-fibra

$$\frac{P_{inject}}{P_{total}} = (NA)^2$$

- DIODE LASER





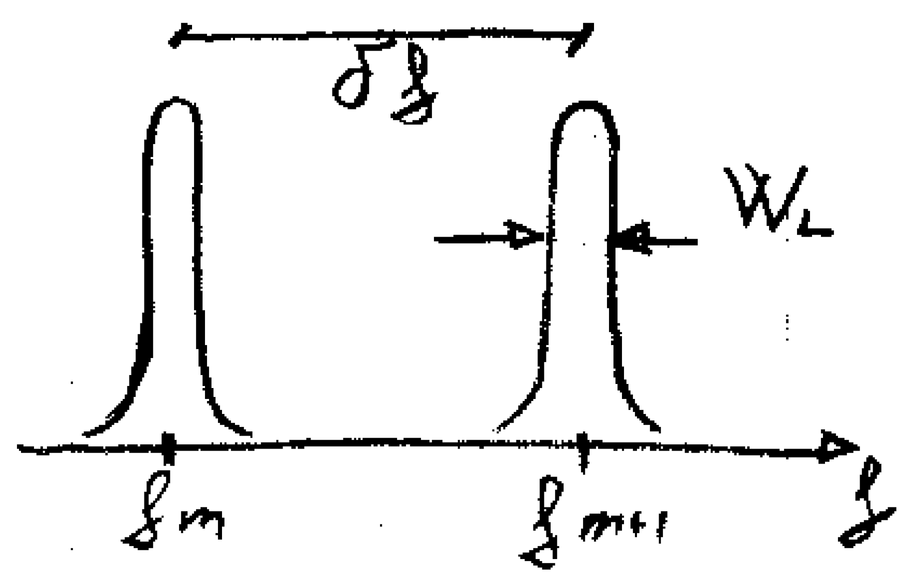
Guany

$$g_m \triangleq a(N - N_0)$$

- Condició d'oscil·lació

$$\boxed{g_n \geq g_{th} = \alpha_s + \frac{1}{2L} \ln \frac{1}{R_1 R_2} = \alpha_t} \qquad \boxed{f_m = m \frac{v}{2L}} \qquad m \in \mathbb{Z}$$

α_s: perdues scatering α_t: perdues totals



$$\boxed{\tau_{ph} \triangleq \frac{1}{v\,\alpha_t}} \qquad W_L \cong \frac{1}{2\pi \tau_{ph}}$$

- Guany net de la llum en el medi

$$g(\lambda) = \Gamma g_m(\lambda) \qquad \Gamma: \text{factor de confinament}$$

$$g_m(\lambda) = a(N - N_0) - \gamma(\lambda - \lambda_p)^2$$

- Condició d'arrencada del LASER

$$\boxed{N > N_{th} \triangleq N_0 + \frac{\alpha_t}{a\Gamma}} \qquad \boxed{I_{th} = \frac{qV}{\tau_r} N_{th}}$$

Densidad de fotones $\boxed{S = (N - N_{th}) \frac{\tau_{ph}}{\tau_{sp}} = (I - I_{th}) \frac{\tau_{ph}}{qV}}$ $\qquad R = \frac{I}{qV} = \frac{N}{\tau_{sp}}$

Si $R_1 = R_2 = R \Rightarrow$ $\boxed{\begin{array}{l} S_{out} = \frac{1}{2} \frac{1-R}{\sqrt{R}} S_0 \\ P_{out} = v\, hf\, Wd\, S_{out} \end{array}}$ $\qquad S_0 \simeq S$

$V = WLd$ (Volum)

$$P_{out} = \frac{1-R}{2\sqrt{R}} \frac{hf}{q\alpha_t L}(I - I_{th})$$

- Modulació digital del LASER

- Cas 1 : $I_{OFF} < I_{th} < I_{ON}$

$$t_D = \tau_r \ln \frac{I_{ON} - I_{OFF}}{I_{ON} - I_{th}}$$

- Cas 2 : $I_{th} < I_{OFF} < I_{ON}$

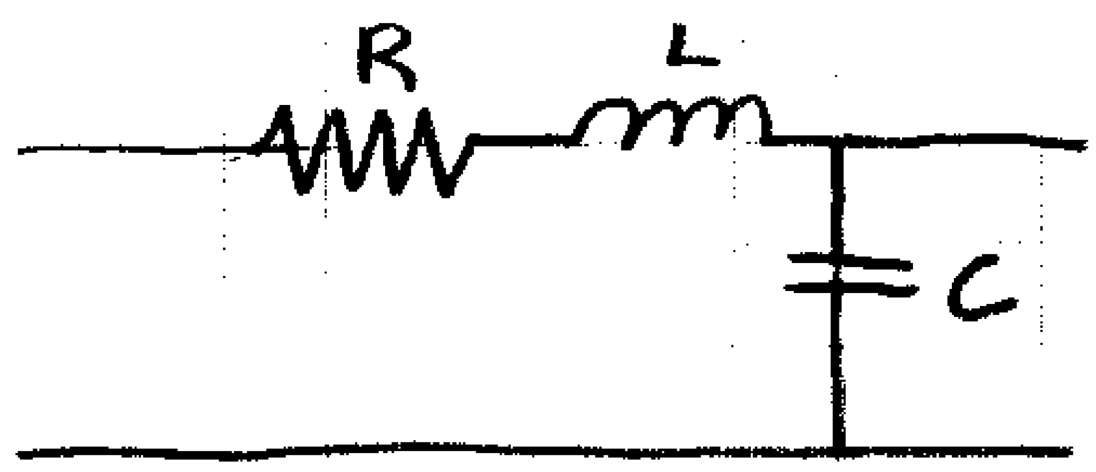
$$t_{ON} = \left[\frac{2qV}{v\Gamma a} \ln\left(\frac{I_{ON} - I_{th}}{I_{OFF} - I_{th}}\right) \frac{1}{I_{ON} - I_{OFF}} \right]^{1/2} \qquad \frac{P_{On}}{P_{OFF}} = \frac{I_{ON} - I_{th}}{I_{OFF} - I_{th}}$$

- Modulación analógica del LASER

$$I(t) = I_0 \, (1 + m \sin \omega_m t)$$

Relajación : $\omega_r^2 \triangleq \frac{v\Gamma a}{qV} (I_0 - I_{th})$

Decaiment : α_0



$$\omega_r^2 \triangleq \frac{1}{LC}$$

$$\frac{\omega_r^2}{2\alpha_0} \triangleq \frac{1}{RC}$$

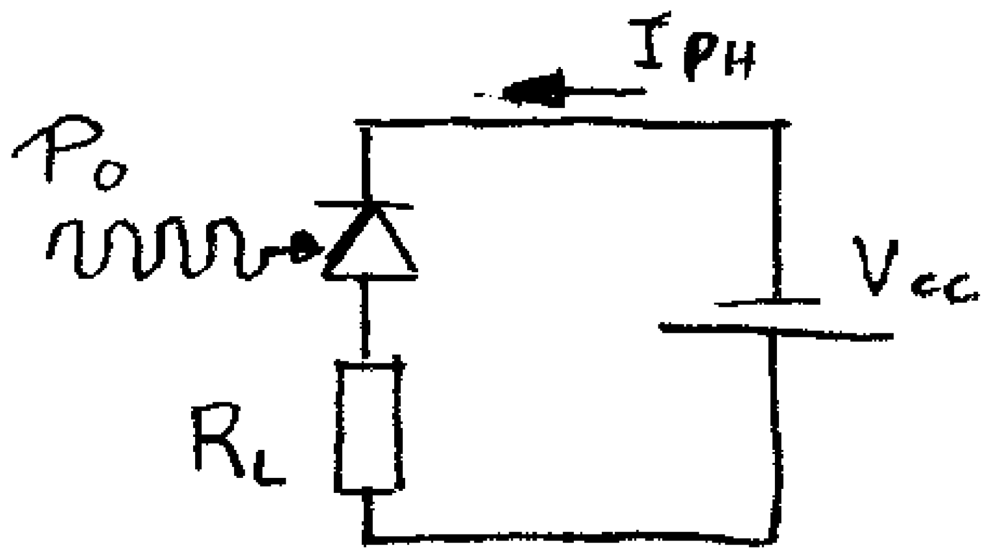
Equivalent elèctric

- Naturalesa aleatoria de la llum

LASER $\begin{cases} \langle m \rangle \\ \sigma_m^2 = \langle m \rangle \end{cases}$ $\qquad SNR \doteq \langle m \rangle$

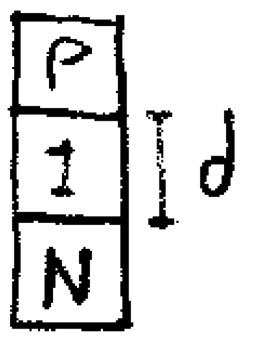
LED $\begin{cases} \langle m \rangle \\ \sigma_m^2 = \langle m \rangle^2 + \langle m \rangle \end{cases}$ $\qquad SNR = \frac{\langle m \rangle^2}{\langle m \rangle^2 + \langle m \rangle} \simeq 1$

1-4 RECEPTORS ÒPTICS



Eficiencia quàntica $\boxed{\eta \triangleq \dfrac{I_{PH}/q}{P_{IN}/h\nu}}$

Responsivitat $\boxed{\mathcal{R} \triangleq \dfrac{I_{PH}}{P_{IN}} = \eta \dfrac{q}{h\nu}}$

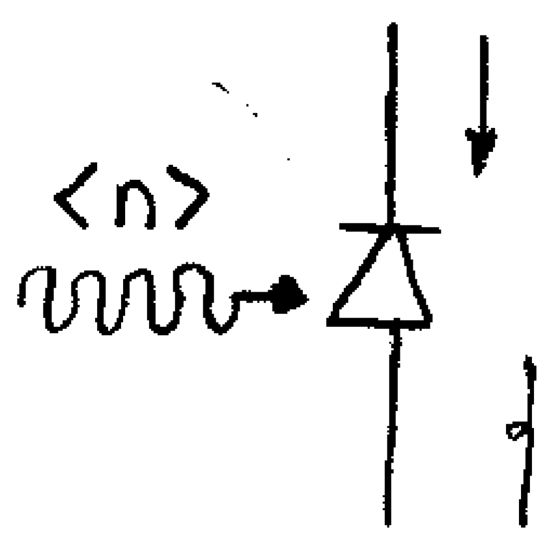


$\eta \triangleq \dfrac{P_{absorbida}}{P_{incident}} \cong (1-R)(1-e^{-\alpha_p d})$ α_p: coef. absorció

- FOTODIODE PIN

$\boxed{i_{PD} = i_{PH} + i_D}$ (fotocorrent) + (corrent de foscor)

∘ Variables normalitzades



B: BW receptor

$\begin{cases} \langle m \rangle \\ \sigma_m^2 \end{cases} + \begin{cases} \langle m_D \rangle \\ \sigma_D^2 \end{cases} + \begin{cases} \langle p \rangle \\ \sigma_p^2 \end{cases}$

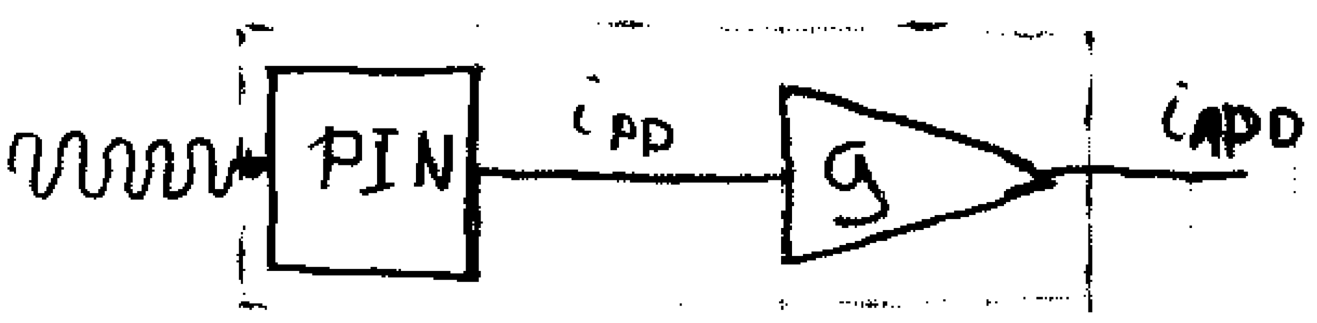
útil, c. foscor, soroll tèrmic

$$\boxed{\begin{aligned} \langle i_{PH} \rangle &\rightarrow \langle m \rangle = \frac{\langle i_{PH} \rangle}{2Bq} \\ \sigma_{PH}^2 &\rightarrow \sigma_m^2 = \frac{\sigma_{PH}^2}{(2Bq)^2} \end{aligned}}$$

$$\begin{cases} I_{PH} \triangleq \langle i_{PH} \rangle = \mathcal{R} \cdot P_{IN} \\ \sigma_{PH}^2 = 2qB \cdot I_{PH} \end{cases} \qquad \begin{cases} \langle m \rangle = \eta \langle n \rangle \\ \sigma_m^2 = \langle m \rangle \end{cases}$$

$$\boxed{SNR^{PIN} = \frac{I_{PH}^2}{2Bq\,(I_{PH} + I_D) + \sigma_{th}^2}} \qquad SNR^{PIN}_{màx} = \frac{\langle i_{PH} \rangle}{2Bq}$$

- FOTODIODE APD



$M = \langle g \rangle$

NF: Figura de soroll

$$\begin{cases} \langle i_{APD} \rangle = M \langle i_{PD} \rangle \\ \sigma_{APD}^2 = M^2\, NF \cdot \sigma_{PD}^2 \end{cases} \qquad \begin{cases} \langle m \rangle_{APD} = M \langle m \rangle \\ \sigma_{m_{APD}}^2 = M^2 \cdot NF \cdot \langle m \rangle \end{cases}$$

$$\boxed{SNR^{APD} = \frac{M^2 I_{PH}^2}{2Bq\, M^2\, NF\, (I_{PH} + I_D) + \sigma_{th}^2}} \qquad SNR^{APD}_{màx} = \frac{\langle i_{PH} \rangle}{2Bq\, NF}$$

- Comparació PIN-APD $(I_D = 0)$

$$SNR_{PIN} = \frac{\langle m \rangle^2}{\langle m \rangle + \sigma_p^2} \qquad SNR_{APD} = \frac{\langle m \rangle^2}{NF\langle m \rangle + \sigma_p^2/M^2}$$

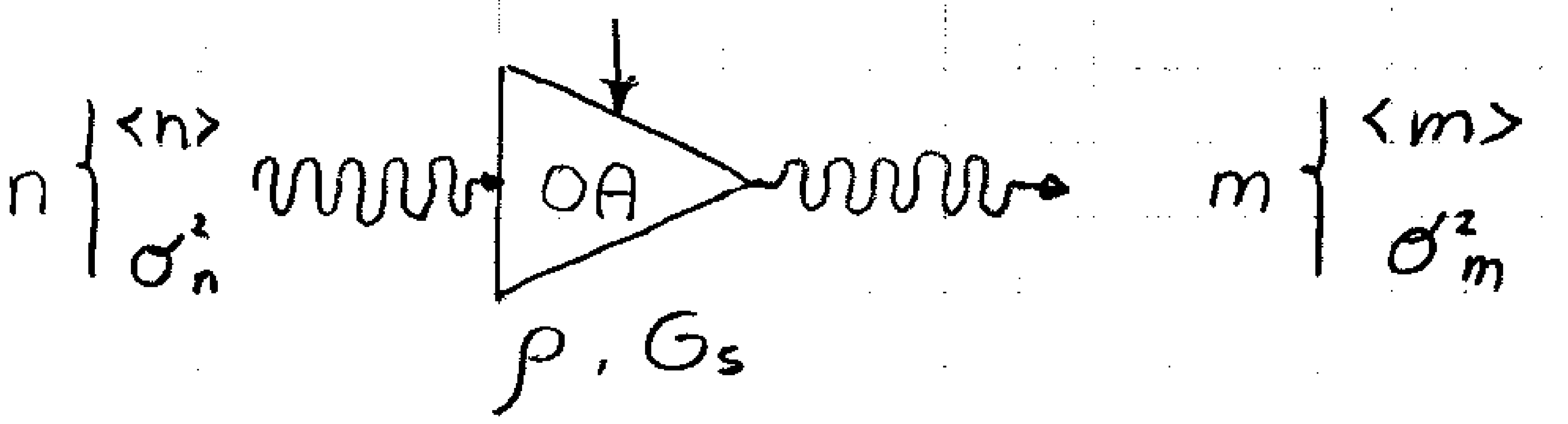
APD mejor PIN $\Leftrightarrow \sigma_p^2 > (F-1)\langle m \rangle$

$$\Rightarrow P_{IN} < \frac{h\nu}{NF \cdot \eta} 2B\sigma_p^2$$

T-5 AMPLIFICADORS ÒPTICS

SOA (Semiconductor Optical Amplifier)

EDFA (Erbium Doped Fiber Amplifier)



ρ : Emisió espontanea

G_s : Guany en passada única

$$\langle m \rangle = G_s \langle n \rangle + (G_s - 1)\rho$$

$$\sigma_m^2 \cong 2\rho G_s^2 \langle n \rangle$$

$$\sigma_m^2 = 2\rho G_s (G_s - 1)\langle n \rangle + \rho (G_s - 1)^2 + G_s^2 (\sigma_n^2 - \langle n \rangle) + G_s \langle n \rangle + \rho (G_s - 1)$$

$$SNR_{IN} = \langle n \rangle$$

$$SNR_{OUT} = \frac{\langle n \rangle}{2\rho}$$

$$NF \simeq 2\rho$$

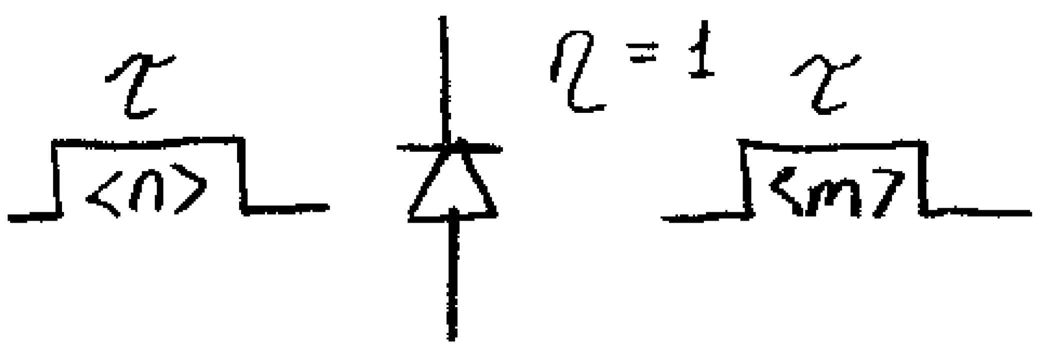
Amplificador Òptic : Funciona igual que un LASER però amb P_{IN} i P_{OUT}

$R_1 = R_2 = R \simeq 10^{-5}$ $\Rightarrow$ Opera per sota de I_{th}

$$G(\lambda) = \frac{G_s (1 - R)^2}{(1 - R G_s)^2 + 4 R G_s \sin^2(2\pi L / \lambda)}$$

1-6 SISTEMES DE TX PER F.O.

- Sistema totalmente ideal



$$P(\epsilon) = \frac{1}{2} e^{-\langle n \rangle}$$

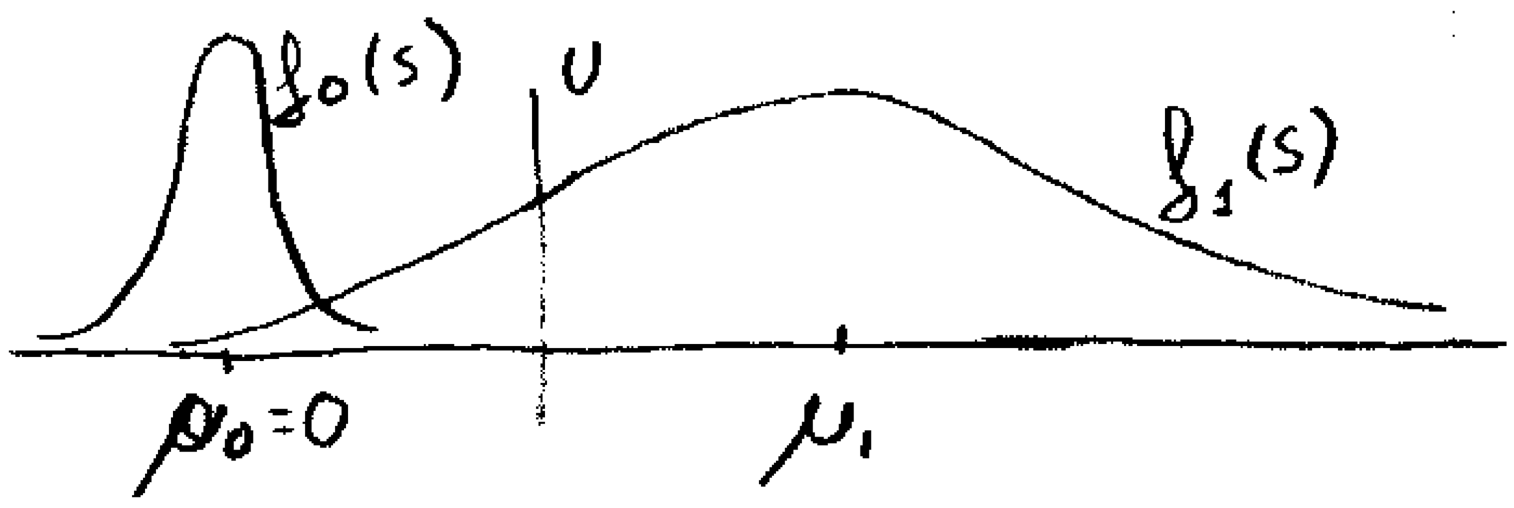
$$\boxed{BER \triangleq \log P(\epsilon) = -0'3 - 0'87 \langle n_a \rangle}$$

$$\langle n_a \rangle = \frac{\langle n \rangle_{bit\,0} + \langle n \rangle_{bit\,1}}{2} \Rightarrow \langle n_a \rangle = \frac{\langle n \rangle}{2}$$

$$\frac{1}{2} e^{-\eta \langle n \rangle}$$

Fotodiodo PIN con $\eta \neq 1$ $\Rightarrow$ $\boxed{BER = -0'3 - 0'87\,\eta \langle n_a \rangle}$

- Cas general



$$\sigma_0, \sigma_1 << |\mu_0 - \mu_1|$$

$$\boxed{\mu_{opt} \simeq \frac{\mu_0 \sigma_1 + \mu_1 \sigma_0}{\sigma_1 + \sigma_0}}$$

$$P_{min}(\epsilon) = \frac{1}{2}\left[1 + \frac{1}{2} erf\left(\frac{u_{opt} - \mu_1}{\sqrt{2}\,\sigma_1}\right) - \frac{1}{2} erf\left(\frac{u_{opt} - \mu_0}{\sqrt{2}\,\sigma_0}\right)\right]$$

- Paràmetres Q

si $u = u_{opt}$ $\Rightarrow$ $\boxed{Q = \frac{\mu_1 - \mu_0}{\sigma_1 + \sigma_0}}$

- 1a Situació : Domina soroll shot $\sigma_1^2 >> \sigma_0^2$

  $SNR \simeq Q^2$

- 2a Situació : Domina soroll tèrmic $\sigma_1^2 \simeq \sigma_0^2$

  $SNR \simeq 4Q^2$

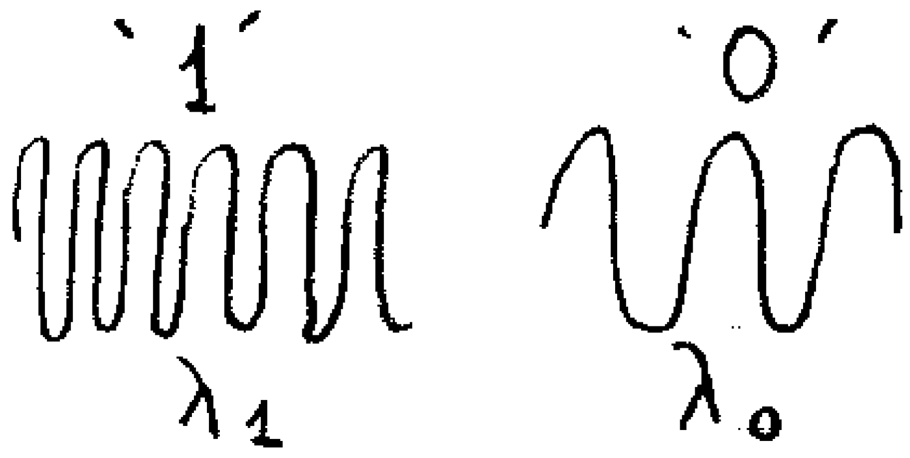
- Cas general

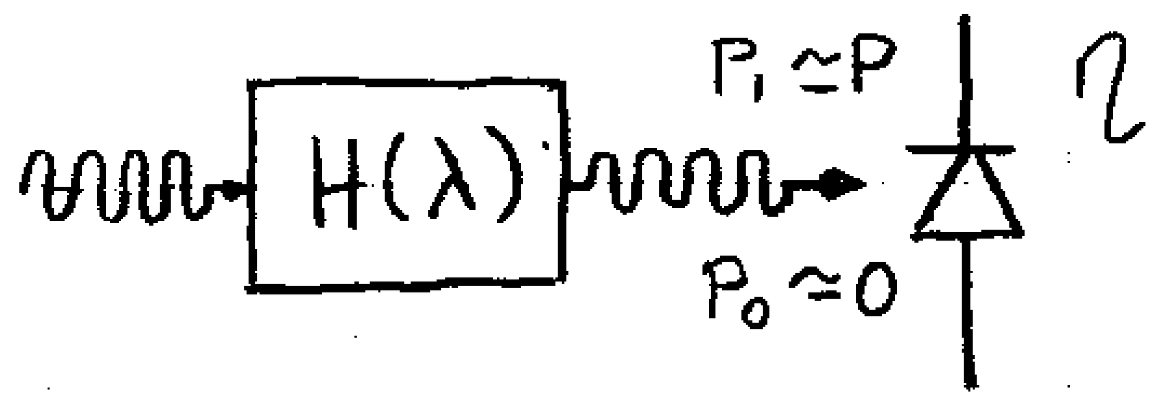
  $$\frac{\sqrt{SNR}}{2} \leq Q \leq \sqrt{SNR}$$

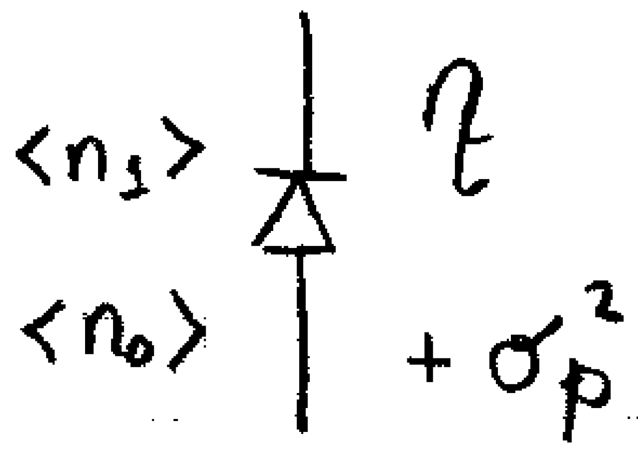
$$\boxed{P_{min}(\epsilon) = \frac{1}{2}\left[1 - erf\frac{Q}{\sqrt{2}}\right] = \frac{1}{2} erfc\frac{Q}{\sqrt{2}}}$$

per $P_{min}(\epsilon) < 10^{-9}$ $\Rightarrow$ $\boxed{Q \geq 6}$

- FSK Òptic







'1' $\begin{cases} \langle m_1 \rangle = \eta \langle n_1 \rangle \\ \sigma_1^2 = \eta \langle n_1 \rangle + \sigma_p^2 \end{cases}$

'0' $\begin{cases} \langle m_0 \rangle = \eta \langle n_0 \rangle \\ \sigma_0^2 = \eta \langle n_0 \rangle + \sigma_p^2 \end{cases}$

$$Q \cong \frac{\eta \langle n_1 \rangle}{\sqrt{\eta \langle n_1 \rangle + \sigma_p^2} + \sqrt{\sigma_p^2}}$$

- Cas 1 : Domina soroll shot $\Rightarrow \eta \langle n_1 \rangle \gg \sigma_p^2$

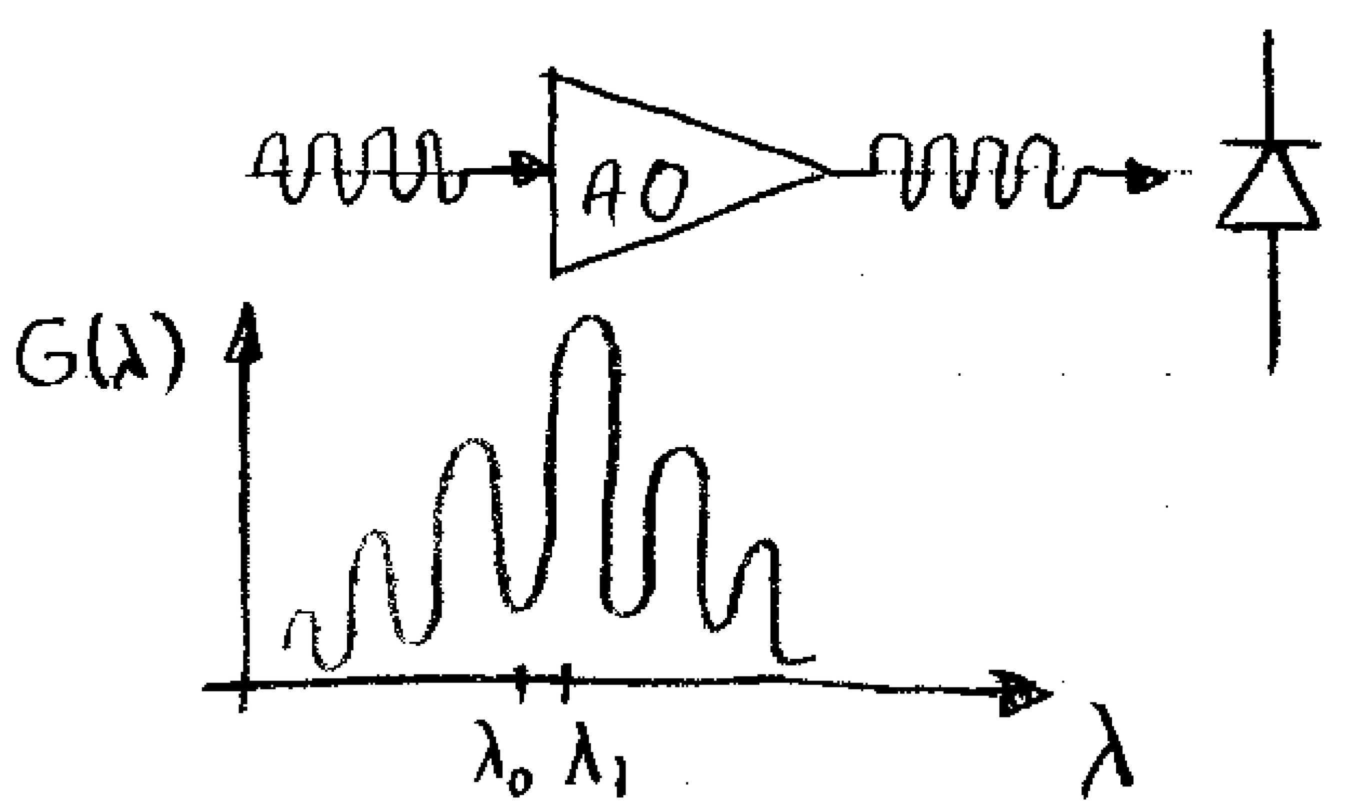
$$Q \simeq \sqrt{\eta \langle n_1 \rangle} \overset{!!}{\geq} 6 \Rightarrow \langle n_a \rangle = \frac{36}{\eta}$$

- Cas 2 : Soroll tèrmic mateix ordre q shot $\Rightarrow \sigma_p^2 \simeq \eta \langle n_1 \rangle$

$$Q \simeq \frac{\sqrt{\eta \langle n_1 \rangle}}{1 + \sqrt{2}} \overset{!!}{\geq} 6 \Rightarrow \langle n_a \rangle = \frac{210}{\eta}$$

- Cas real (intermitg) $\Rightarrow \frac{36}{\eta} < \langle n_a \rangle < \frac{210}{\eta}$

- FSK òptic + PAO



$\langle n_a \rangle \geq 72\rho$

- Model estadístic d'un AO (Ampliación)

$$\begin{cases} \langle m \rangle = G_s \langle n \rangle + \rho (G_s - 1) \\ \sigma_m^2 = 2\rho G_s (G_s - 1)\langle n \rangle + \rho^2 (G_s-1)^2 + G_s^2(\sigma_n^2 - \langle n \rangle) + G_s \langle n \rangle + \rho(G_s - 1) \end{cases}$$

En general $\sigma_n^2 > n \Rightarrow \sigma_n^2 = \alpha \langle n \rangle$

$SNR_{IN} = \dfrac{\langle n \rangle}{\alpha}$ $\quad SNR_{OUT} = \dfrac{\langle n \rangle}{\alpha - 1 + 2\rho}$ $\quad NF = \dfrac{\alpha - 1 + 2\rho}{\alpha}$

- Cas particular : Fibra òptica $\rho = 0$; $G_s = A < 1$

$$\begin{cases} \langle m \rangle = A \langle n \rangle \\ \sigma_m^2 = A^2(\sigma_n^2 - \langle n \rangle) + A \langle n \rangle = A \langle n \rangle = \langle m \rangle \end{cases}$$

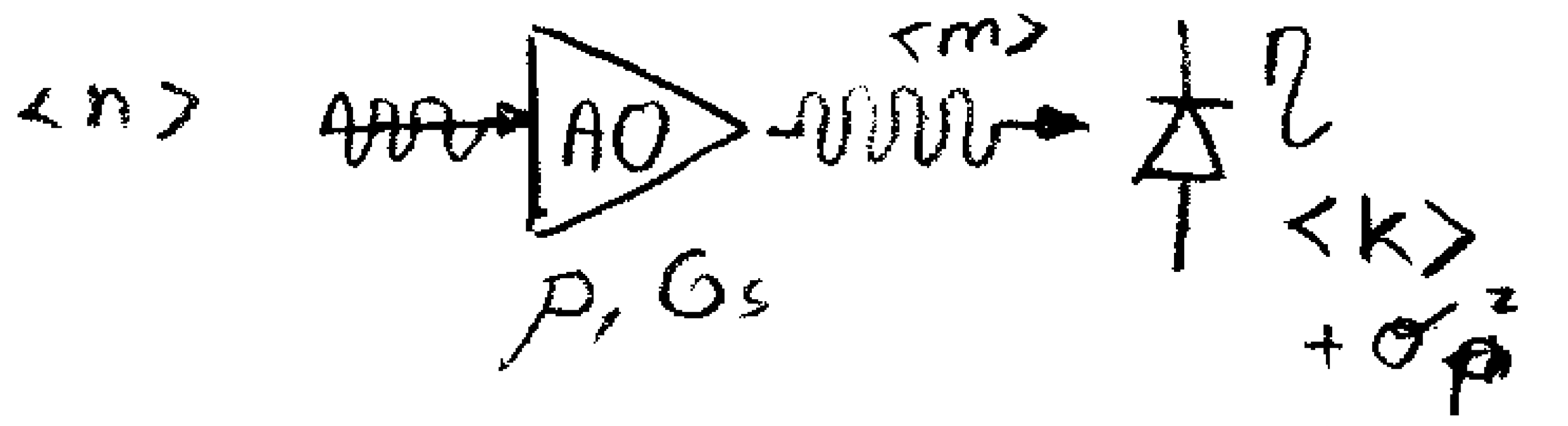
(llum coherent $\sigma_n^2 = \langle n \rangle$)

$SNR_{IN} = \langle n \rangle$ $\quad SNR_{OUT} = A \langle n \rangle$ $\quad NF = 1/A$

- Cas particular : Fotodiode $\rho = 0$; $G_s = \eta$

Idem Fibra amb $A = \eta$

- Cas particular : Preamplificador + Fotodiode



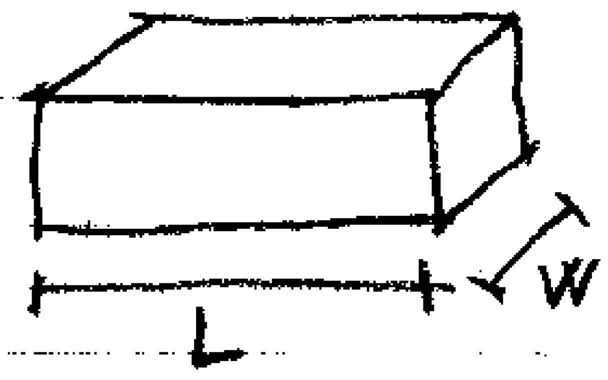
$$\begin{cases} \langle K \rangle \simeq \eta G_s \langle n \rangle \\ \sigma_K^2 \simeq 2\rho G_s^2 \eta^2 \langle n \rangle + \eta G_s \langle n \rangle \end{cases}$$

$SNR_{IN} = \langle n \rangle$

Si $G_s^2 >> \dfrac{\sigma_P^2}{2\rho\eta^2\langle n \rangle}$ $\Rightarrow SNR_{OUT} = \dfrac{\langle n \rangle}{2\rho}$ $\quad NF = 2\rho$

$\Rightarrow$ Tenim la mateixa SNR que només amb el AO

- Reflectividades R_1, R_2, $R_1 = R_2 = 0$
- Nivel de transparencia N_0
- Paràmetro de emisión espontanea β
- Factor de confinamiento Γ
- Perdidas scattering α_s
- Corriente umbral I_{th}



Ecs. ritmo Laser:

Portadors $$\frac{dN}{dt} = \frac{I}{q \cdot V} - \frac{N}{\tau_r} - v \cdot g\, S$$

$J = I \cdot W L$

Fotons $$\frac{dS}{dt} = \text{~~} v \cdot g \cdot S - v \cdot \alpha_t S \text{~~} \; v(g - \alpha_t)\, S + \beta \frac{N}{\tau_r}$$

- N densitat portadors zona activa
- τ_r temps de vida del portador
- g guany per emisió estimulada neta
- α_t perdues totals en la cavitat
- τ_p temps permanencia fotó a la cavitat
- S densitat de fotons

- a coeficient de guany
- γ factor de curvatura
- g_p guany màxim
- g_n guany net de la llum al medi
- g_m guany del material

- I_d corriente oscuridad media
- I_{PH} fotocorriente media
- I_{TH} corriente de ruido termico media

$$\langle i_{th} \rangle = 0$$
$$\sigma_{th}^2 = \langle i_{TH}^2 \rangle = 4 \frac{KTB}{R_L} F_A$$

$$N_{th} = N_0 + \frac{\alpha_t}{\Gamma a}$$

$$B_e = \frac{V_t}{2} \qquad \sigma \cdot B_c = \frac{0'1874}{\sqrt{2}}$$